Bridge dynamics and dynamic amplification factors — a review of analytical and experimental findings

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The dynamic amplification factor (DAF) is an important parameter in the design of highway bridges and yet no worldwide consensus has been reached so far as to its value. Some disagreement exists between provisions of various national bridge codes. This is because the DAF depends, in addition to the maximum span or the natural frequency, on many other parameters that are difficult to take into account with reasonable accuracy. Vehicle speed, weight, and dynamic characteristics, the state of the structure, roadway roughness, expansion joints, the type of bridge supports, soil–structure interaction, and influence of secondary elements are some aspects influencing the DAF. This study reviews the analytical and experimental findings on bridge dynamics and the evaluation of the DAF.

Key words: bridges, vibrations, bridge testing, bridge design codes, dynamic amplification factor.

Le facteur d'amplification dynamique (FAD) est un paramètre important dans la conception des ponts routiers. Pourtant il n'existe, jusqu'à maintenant, aucun consensus international quant à sa valeur, et certaines divergences existent entre les recommandations de différentes normes nationales de calcul de ponts. Ceci est dû au fait que le FAD dépend, en plus de la portée et de la fréquence propre, de plusieurs autres paramètres qui sont difficiles à isoler. La vitesse, le poids et les caractéristiques dynamiques du véhicule, l'état de la structure, l'étendue des imperfections de la planéité de la chaussée, l'état des joints de dilatation, les types d'appuis, l'interaction sol-structure, l'influence des éléments secondaires, sont quelques aspects influençant le FAD. Cette étude présente une revue des recherches théoriques et expérimentales sur la dynamique des ponts et l'évaluation du FAD.

Mots clés : ponts, vibrations, essais des ponts, normes de calcul des ponts, facteur d'amplification dynamique.

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Introduction

The dynamic character of the response of bridges to traffic is presently well established. A moving vehicle on a bridge generates deflections and stresses in the structure that are generally greater than those caused by the same vehicle loads applied statically. The dynamic amplification, (DA) resulting from the passage of one vehicle on a specific bridge is

$$[1] \quad \mathrm{DA} = \frac{R_{\mathrm{dyn}} - R_{\mathrm{sta}}}{R_{\mathrm{sta}}}$$

where R_{dyn} is the maximum dynamic response of the bridge and R_{sta} is the maximum static response. Therefore,

$$[2] \quad R_{\mathrm{dyn}} = R_{\mathrm{sta}}(1 + \mathrm{DA})$$

where (1 + DA) is the dynamic amplification factor (DAF) for the structure. For example, a DAF value of 1.3 corresponds to a dynamic amplification of 30%. As will be seen later, there are many ways of interpreting this simple and straightforward definition of the DAF from test data. In most codes, for design purposes, the dynamic effects of all vehicles on all bridges are taken into account by multiplying the static live loads by a dynamic load allowance (DLA) greater than 1.

In the load carrying capacity evaluation of an existing bridge, three alternatives are available to the structural engineer to obtain the DAF, i.e., (*i*) to use the DLA given by codes, (*ii*) to perform complete dynamic analyses, and (*iii*) to test under controlled or normal traffic loads.

The DLA as given by codes is generally a function of the span of the bridge. However, more recently, a DLA related to the natural frequency of the bridge has been favored (OHBDC 1983*a*, 1983*b*; CAN/CSA-S6 1988). This factor is reliable, but does not yet account for such parameters as the speed, weight, and dynamic characteristics of the vehicle, the type of bridge and state of the structure, damping characteristics, irregularities of riding surface and expansion joints, bridge supports, soil–structure interaction, and influence of secondary elements.

Although specified live loads are different in most countries, this does not explain the large disagreement on the DLA for different national codes that can be seen in Fig. 1. This figure shows the variation of the DLA, as recommended by American (AASHTO 1989), Swiss (SIA 160E 1988), British (BS 5400 1978), Ontarian (OHBDC 1983*a*, 1983*b*), and Canadian (CAN/CSA-S6 1988) standards. Also shown are the DLA for India, Germany, and France, as reported by Coussy *et al.* (1989). All DLAs are plotted versus natural frequency, including those relations given originally in terms of maximum span. The frequency, f_0, has been related to the maximum span, $L_{\max}$, by the following correlation (Tilly 1986):

$$[3] \quad f_0 = 82L_{\max}^{-0.9}$$

This correlation was suggested by RILEM Committee 65 MDB and is based on testings performed on more than 200 European bridges. Other similar relations, $f_0 = f(L_{\max})$, have been reported in the literature (Cantieni 1983; Billing 1984).

A complete dynamic analysis of the bridge–vehicle interaction problem can be done. However, this requires knowledge of the many parameters stated above. In addition, such an analysis would have to be carried out for a large number

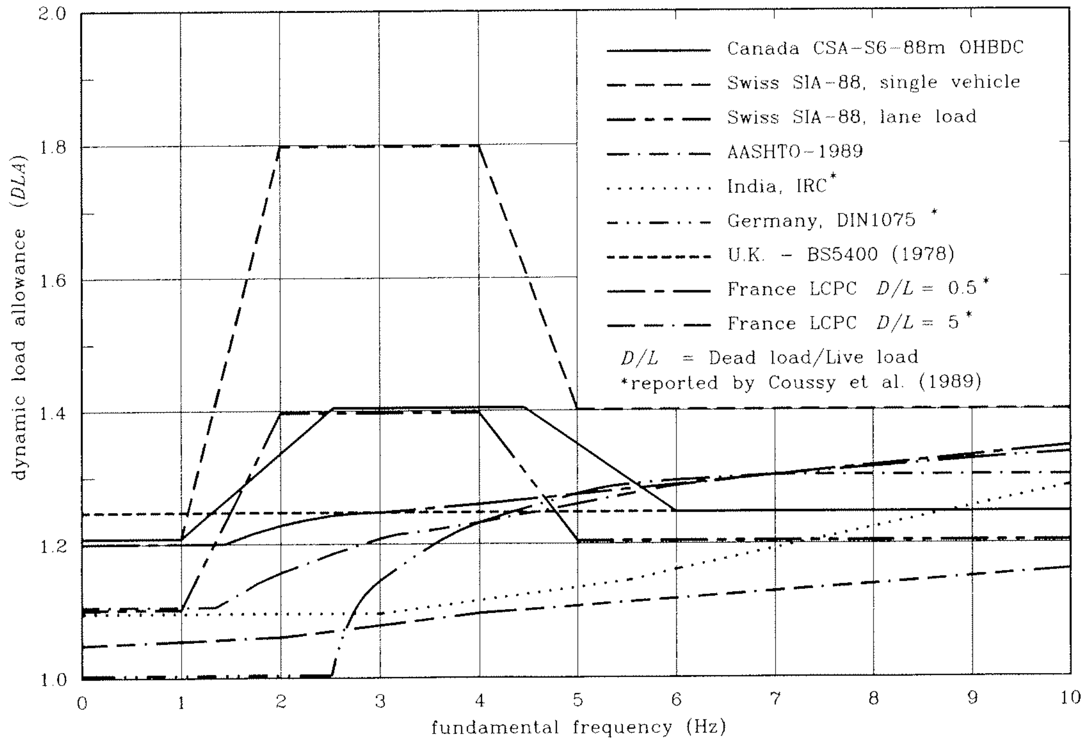

FIG. 1. Dynamic load allowance (DLA) versus fundamental frequency for different national codes.

of vehicles with different dynamic characteristics. This is clearly prohibitive at the present time.

The use of full-scale dynamic testing under controlled or normal traffic conditions remains the most reliable and cost-effective way of obtaining the DAF value for a specific bridge. The increasing interest in these tests is attributed to affordable data acquisition systems and the development of automatic real-time signal processing hardware. Full-scale dynamic testings on bridges are useful for two reasons:

1. They yield an actual instantaneous evaluation of the dynamic properties of the structure (frequency, damping, stiffness) using identification methods. They are also useful in quantifying the extent, evolution, and rate of bridge degradations through behavioural "signatures" of consecutive tests, allowing thereby an effective predictive maintenance.

2. They allow an evaluation of the DAF in terms of the natural frequency of the structure. The relevance of this arises from the fact that the more accurate the DAF, the better the control of the costs of repairs.

Numerous experimental procedures have been used during the last decades. These techniques generally fall in one of the following categories: (*i*) impact tests, (*ii*) use of eccentric or reactional mass exciters, (*iii*) use of test vehicles, i.e., controlled traffic, (*iv*) use of normal traffic, (*v*) wind-generated (ambient) vibrations, (*vi*) sudden release of static load or imposed displacement, and (*vii*) vibrations generated by braking vehicles. All these procedures have been used more or less successfully for the evaluation of the dynamic characteristics of bridges. However, the DAF can only be evaluated under controlled and normal traffic conditions (*iii*) and (*iv*).

This paper first presents a review of the theoretical and experimental findings on dynamics of bridges. Then, some of the more important parameters in bridge dynamics that have an influence on the DAF are discussed in detail. Finally, different methods for computing the DAF from the recorded test data are presented.

Dynamics of bridges — a review

The problem of dynamic amplification was recognized in the 19th century (Cantieni 1983; Fleming and Romualdi 1961). Following the collapse of some railway bridges in Great Britain, Willis (1849) conducted laboratory tests on cast iron beam models. However, it is only recently that a major effort has been directed toward the problem of highway bridge dynamic amplifications. The first important report on this subject was published in 1931 by a special committee of the American Society of Civil Engineers (ASCE 1931). The recommendations of this committee were based on the data obtained from a series of field tests and formed the basis for American design specifications. In particular, it was found that bridge decks had different response characteristics than main longitudinal members. Different dynamic amplification factors for these components were therefore suggested.

It appears from the literature review that most dynamic tests performed on bridges in the early part of this century were designed to yield dynamic properties and very few were aimed at the evaluation of the dynamic amplification factor. Wright and Green (1959) presented a critical review of previous studies related to the vibration of simple span bridges loaded with sprung moving loads. They also reviewed the early field test results reported by Vandegrift (1944), Norman (1950), Foster (1952), Mitchell (1954), Hayes and Sbarounis (1955), Foster and Oehler (1955), Edgerton and Beecroft (1955*a*, 1955*b*), Suer (1955), Biggs and Suer (1955), Biggs *et al.* (1956), Oehler (1957), and Biggs

(1959). They found that the design specifications at the time which recommended limiting deflections and proportions of spans did not always result in designs preventing large vibrations under normal traffic loading. They also suggested the use of normal commercial vehicles for field tests of highway bridges. This report formed the first part of an investigation conducted as part of the Ontario Joint Highway Research Programme. The second part of this investigation consisted in a study of human sensitivity to vibrations; and the third part, reviewed below, consisted in field studies on bridge–vehicle interaction.

Fleming and Romualdi (1961) presented an analytical study of the dynamic responses of single-span and three-span continuous bridges to transient loads. A model beam was constructed to evaluate the validity of a proposed mathematical model for bridges and vehicles. The effects of many parameters on the dynamic amplification factor were then studied and the results compared with the AASHTO recommendations. It was found that for small spans (<13 m), the DAF was higher than that predicted by AASHTO, whereas for medium spans (>13 m), the DAF was in accordance with the AASHTO recommendations. The following conclusions were reached:

1. For unsprung loads, the velocity of the load is the most important variable. Modifying the mass of the load changes the velocity at which the maximum dynamic amplification occurs.

2. A large dynamic amplification can be produced in the bridge due to the initial vibration of the vehicle caused by unevenness of the bridge approach.

The American Association of State Highway Officials conceived and sponsored a major experimental investigation (AASHO 1962). The main objectives were to (*i*) determine the behaviour of short span highway bridges under repeated applications of overstress, and (*ii*) determine the dynamic effects of moving vehicles on these bridges. Eighteen bridges were built for these purposes. Each bridge was a simple span structure consisting of three 15 m beams and a reinforced concrete slab. The beams in ten bridges were fabricated from wide-flange rolled steel sections; eight were independent of the slab and two were connected to the slab with channel connectors. The beams in four bridges were built with high-strength concrete prestressed beams. The remaining four bridges were of "cast-in-place" T-beams construction. The project started in 1956, and the testing began in 1958, to be completed in 1961.

To evaluate the dynamic effects of moving vehicles on the response of the bridges, test vehicles (two-axle trucks and three-axle tractor–semitrailor combinations) made approximately 1900 runs over 15 bridges at speeds varying between 12 and 32 km/h. The following observations were made:

1. Spring models could be used with reasonable accuracy to represent the static load–deflection characteristics of the vehicle, including the effect of frictional damping in the suspension system. Viscous damping in the tires was found to be close to 1%.

2. Initial vertical oscillations of the vehicle were present in practically all tests and introduced a large uncertainty in the dynamic response of the bridges.

3. The DAF generally increased with the speed parameter, $S_p = vT/2L$, where v is the speed of the vehicle, L is the span length, and T is the fundamental period of the bridge. The largest value of the DAF for displacement was 1.63, and only 5% of the measured DAF exceeded 1.40. The largest value of the DAF for moments (strains) was 1.41 and only 5% of the measured DAF exceeded 1.286, the value specified by the impact formula in the AASHO standard specification for highway bridges at the time.

4. The peak value of the DAF was not strongly influenced by the weight ratio (the ratio of the gross weight of the vehicle over the total weight of the bridge).

5. The DAF increased with the number of vehicle runs during regular traffic only for bridges with permanent set and approach profile roughness increasing with time. A decrease in the DAF was measured for bridges with a decrease in camber and a smoothing of the approach profile by patching.

6. Blocking of vehicle springs resulted in approximately doubling the DAF values.

7. The interleaf friction in the suspension system of the trucks has an important effect on the dynamic response of the bridge.

Walker and Veletsos (1963) presented the results of an analytical investigation of the response of single-span highway bridges to moving vehicles. The bridge was idealized as a single span but divided into several damped masses. The vehicle was represented as a multi-axle sprung load and the following vehicle parameters were studied: initial vehicle oscillation, interleaf friction, damping of the suspension system, vehicle speed, mass of vehicle and its axle spacing. Bridge type, bridge roughness, sag and camber and bridge damping were studied. The following significant parameters were discussed in the study:

1. The speed parameter (S_p, defined above): For an initially vibrating vehicle, the speed parameter controls both the amplitude of the response component, corresponding to the moving constant force effect, and the location of the vehicle on the span for which the interacting force attains its maximum value.

2. The axle spacing parameter: This parameter, which depends on the speed parameter, determines whether the components of the response due to each axle add or cancel. It particularly affects the value of the maximum static response and therefore the DAF.

3. The weight ratio (defined above) and the frequency ratio (taken as the ratio of the vehicle and bridge vibration frequencies): These two parameters were developed to describe the basic characteristics of mass and stiffness of the bridge–vehicle system and can considerably influence its dynamic response if taken separately. However, since these ratios are interrelated, the system response is generally insensitive to their variations.

4. The vehicle suspension parameters: These parameters define the frictional characteristics of the vehicle suspension, and are significant in the event of a vehicle having large amplitudes of initial oscillation.

5. Initial conditions of the vehicle: For the conditions that were considered in this report, there was an almost linear relationship between the amplitude of the initial motion of the vehicle and the maximum response of the bridge.

Wright and Green (1963) presented the results of an extensive Ontario program of field studies of bridge–vehicle interaction conducted on 52 highway bridges, in 1956 and 1957. The following characteristics were determined for each bridge: (*i*) span length and structural arrangement, (*ii*) dead weight, (*iii*) calculated stiffness, (*iv*) calculated natural frequency, and (*v*) deflection under live load. Dynamic motion recordings were made for each bridge under controlled and normal traffic conditions. Experimental values of stiffness, frequency, dynamic amplification factor, and damping were derived for the structures. Some of the principal findings of this research program were the following:

1. Theoretical studies on bridge–vehicle interaction available at the time were of little use for design purposes.
2. Experimental values of the stiffness were always larger than the calculated values, because of the parameters not ordinarily considered in design calculations (composite action, joint rigidity, participation of secondary elements, etc.).
3. Natural frequencies of simply supported spans could be predicted with reasonable accuracy, although some of the experimental values tended to be larger than the calculated values. A substantial number of forced vibrations associated with the natural frequency of the test vehicle suspension systems (at about 2.5 Hz) were observed.
4. Dynamic amplifications of 30% were typically observed, with peak values of up to 75%.
5. The DAF was strongly affected by irregularities found in the bridge approaches, by the smoothness of the riding surface (bumps, expansion joints, etc.), and by the application of vehicle brakes.
6. The DAF values were much larger for bridges with low stiffness.

The authors recommended that attempts should be made to eliminate irregularities in the deck and approach pavements and over bearings, as these factors were found to be much more important than bridge strength, stiffness, and geometry. They also recommended that further studies on the influence of roadway irregularities on bridge vibration should be undertaken.

Tung (1967) formulated the problem of highway bridge response to traffic on a probabilistic basis recognizing thereby that vehicular traffic is statistical in nature. A simple Poisson process was used to model traffic flow pattern and the response was evaluated using probability density functions and a linear damped system for the bridge.

Smith (1969) presented a study on the dynamic behaviour of highway bridge structures as part of the Ontario Joint Highway Research Programme. This report was concerned with current knowledge on the dynamic amplification factor, fatigue in bridges, and human sensitivity to vibrations. Significant parameters influencing the DAF were set forth: span length, vehicle speed, axle spacing, and fundamental frequency.

Veletsos (1970) presented a numerical method developed at the University of Illinois for the computation of dynamic response of highway bridges to moving vehicles. The method was applied to cantilever bridges, using a linear elastic beam model with distributed flexibility and lumped masses. The vehicle was idealized as a three-axle sprung load model with proper damping characteristics for the suspension system.

Whitmore (1970) studied the dynamic effects of heavy vehicles moving on pavement roadway. Roadway profile irregularities (using a profilometer and a spectral density technique) and vehicle characteristics (mass distribution, suspension system, speed) were examined. The objectives were (*i*) to investigate the effect of roadway profile and vehicle characteristics on the dynamic loadings, (*ii*) to isolate specific parameters influencing loads transmitted to the pavement, and (*iii*) to develop a technique capable of predicting the dynamic loads from the aforementioned parameters. The following observations were made:

1. In the presence of step bump irregularities, higher values of the DAF were observed as well as a wide range of frequency components in the dynamic motion.
2. The force at the tire–roadway interface could be measured on the vehicle or on the roadway, and was found to increase with tire pressure and suspended mass.
3. Damping of the dynamic loads dependend on the vehicle mass and suspension system.

Csagoly *et al.* (1972) presented the results from a second series of field tests in Ontario (the first one reviewed above, see Wright and Green) carried out on eleven bridges. The frequencies obtained from a computer program were compared with the measured values obtained from these tests. Some of the conclusions reached were as follows:

1. The dynamic analysis program yielded frequencies acceptable in engineering practice.
2. The first and second modes of vibration should have frequencies lower than 2 Hz and higher than 5 Hz respectively to limit excessive vibrations. This conclusion was particularly important, as this range of values corresponds to the fundamental frequencies of typical commercial vehicles.
3. Dynamic amplification is a function of roadway roughness and bridge deck conditions.

Leonard (1974) presented the techniques and instrumentation used in full-scale dynamic tests on highway bridges at the Transport and Road Research Laboratory (TRRL) of the United Kingdom. The test program had the following objectives; (*i*) to measure the peak levels of vibration and the frequencies for road and foot bridges under normal conditions of use; (*ii*) to determine the natural frequencies, mode shapes, damping and static stiffness; (*iii*) to relate the response of bridges to the dynamic behaviour of vehicles and to the excitation by pedestrians; and (*iv*) to investigate the influence of variations in structural parameters on dynamic response.

Deflections and accelerations were measured for dynamic loading generated by a mobile excitation system, the energy input device (EID). Traffic-induced forces could not be measured or controlled in a practical way at the time. The EID, consisting of four masses mounted on a mobile axle and driven by an electro-hydraulic actuator, addressed these shortcomings by applying a force at a predetermined frequency. However, this method does not represent the normal vehicle–bridge interaction and was developed to examine the response to a moving oscillatory load. It was found that most of the bridges considered had a fundamental frequency below 5 Hz, the resonating frequency of the EID on its tires, and that this method could provide a practical means of comparing the behaviour of different bridges.

Eyre (1976) conducted five dynamic tests on the main span (214 m) of the Cleddau Bridge at Milford Haven, United Kingdom, having a total length of 820 m and consisting of a steel box girder and a concrete deck. The main objective of the tests was to determine the dynamic characteristics of the bridge, which was excited by the sudden release of a 32.2 metric ton weight from a cable attached to the superstructure at the centre of the span. The resulting dynamic response was recorded using gauges fixed at different positions on the superstructure and piers. A limited number of vibration measurements were also taken under normal traffic loads and ambient wind loads. The following observations emerged from this study:

1. The damping values were measured as an average form of logarithmic decrement and were found to be 0.043–0.061 at a fundamental frequency of 0.53 Hz.
2. The measured values of damping were constant for amplitudes of deflection of up to ±13 mm.
3. Good agreement was found between amplitudes of vibration on the side spans relative to the centre span and modal analysis predictions.
4. Traffic-induced vibrations were not significant at a higher frequency than at the fundamental frequency of the bridge.

Eyre and Tilly (1977) focused their interest on the measurement of damping properties of bridges. Measurements were taken on 23 bridges having spans of 17–213 m, consisting of steel box girders with steel decks, steel box girders with concrete decks, and steel plate girders with concrete decks. The authors found that the measured damping values increased with the amplitude of vibration and could reach values four times higher than the values for small amplitudes. They also noticed a tendency for damping to increase with frequency, resulting in higher values of damping for single-span bridges than multi-span bridges.

Campbell *et al.* (1977) carried out further studies on the test data obtained from the second series of bridge testing in Ontario (see Csagoly *et al.* above). One of the objectives of this investigation was to compare the DAF values obtained from measurements with those recommended by the AASHTO design specifications. They concluded that (*i*) the DAF based on tests was generally higher than that of AASHTO for bridges having their natural frequencies ranging from 2.5 to 4.5 Hz; (*ii*) the beam idealizations used to predict fundamental frequencies were sufficiently accurate for design purposes; and (*iii*) the dynamic response was dependent on the exciting frequency of the vehicle with respect to the fundamental frequency of the bridge.

Shepherd and Sidwell (1977) reported the results of analytical and experimental investigations on the dynamic characteristics and responses of five prototype concrete bridges having 3 or 4 spans varying from 13 to 38 m. Two sources of vibration were used: a dynamic exciter positioned at the centre of one of the internal spans and a test vehicle driven across the bridges at a speed varying from 12 to 88 km/h. It was found that damping values corresponding to the first horizontal mode exceeded those corresponding to the first vertical mode. This was explained by the fact that for horizontal excitation, the piers and abutments moving within the ground absorb a large amount of energy. The first vibration frequencies were predicted with reasonable accuracy, using a beam model with lumped masses for the bridges.

Moses (1979) developed a procedure where instrumented bridges were used to evaluate the axle loads and the total weight of the vehicles in motion, using strain gauges and timing switches. Information collected by this type of field test would be very useful in bridge maintenance and rehabilitation. It was found that the system produced repeatable output and could predict vehicle weights in agreement with nearby truck weigh stations.

Gupta and Traill-Nash (1980) investigated the vehicle braking effects on bridge response. For a bridge subjected to traffic loading, a redistribution of the axle loads can occur if a vehicle brakes or accelerates. Single-span, multigirder highway bridges were considered and represented by three mathematical models: (*i*) a beam representation; (*ii*) a beam with torsional freedom; and (*iii*) a uniform orthotropic plate allowing for eccentric loads to be taken into account. A two-axle sprung mass system with frictional damping was used for the vehicle. The authors presented DAF curves and compared the absolute maximum values of the DAF for constant uniform speed and breaking effects. They showed that a nonuniform vehicle motion can considerably influence the dynamic response of a bridge. They also discussed the validity of the three mathematical models used for the bridges and the effects of excentric loading. The effects of initial bounce were also studied by Gupta (1980), using the same mathematical models, with similar conclusions.

In 1981, the ASCE Committee on Loads and Forces on Bridges endorsed the current AASHTO practice as far as live load impact is concerned (ASCE 1981). The Committee recommended, however, that the word "impact" be replaced wherever appropriate in current design specifications by the more descriptive words "dynamic allowance for traffic loadings." The Committee further recommended that other national codes in effect worldwide, and particularly the 1979 Ontario Bridge Code, should be evaluated and compared with the AASHTO specifications before additional changes are to be considered.

Heins and Lee (1981) presented the experimental results obtained from the static and vehicle-induced dynamic field testings of a two-span continuous-curved composite steel box girder, located in Seoul, Korea. The bridge response to test vehicle loading was measured by strain and deflection gauges placed at various locations along the span. The experimental and analytical results using different well-known finite element programs were compared and good agreement was observed.

Douglas *et al.* (1981) conducted a series of dynamic tests on a highway bridge by using the pullback and quick-release method of excitation in order to determine the fundamental frequencies and the transverse natural modes. Hydraulic rams at every pier were used to deform the structure statically and the fluid was released to generate free vibrations. The bridge was also analyzed using a finite element program and the results were treated using system identification methods. The results indicated that the soil–structure interaction phenomenon was more influenced by the overall rotations of the pile foundations than by the lateral stiffness of the pile groups.

Kostem *et al.* (1981) presented the results of a full-scale dynamic testing of a major steel truss highway bridge (630 m) submitted to vehicle loading. The main objective of these was to determine the bridge degradation due to fatigue. A lumped-mass computer model for the bridge and the moving point loads for the vehicle were used to simulate the dynamic behaviour of the system. Some of the conclusions drawn from this study were the following:

1. The dynamic behaviour of long-span truss bridges is dominated by their fundamental modes of vibration.
2. The dynamic response of bridge superstructures can be considerably affected by a modification in the dynamic soil–structure interaction.
3. An increase in static indeterminacy reduces the peak stresses and displacements. However, this increases the amplitude and the duration of low-level vibrations.
4. Vehicle characteristics and roadway roughness were

found to have little effect on the dynamic response of long-span bridges.

Pardoen *et al.* (1981) reported the difficulties encountered in identifying the modal characteristics of the steel truss Toe Toe Stream Bridge (New Zealand). Accelerations were recorded during dynamic tests carried out with a shaker. The resulting motion was difficult to separate into clear transverse, vertical, longitudinal, or torsional modes, partly because of the fact that too few instruments were used relying on a misleading structural symmetry.

Honda *et al.* (1982) derived the power spectral density (PSD) of road surface roughness on 56 highway bridges, measured using a surveyor's level. For each bridge, 84 lines at 10–20 cm intervals and 0.5 and 2.0 m from the centerline of the road were measured. The authors observed that the PSD of roadway roughness can be approximated by an exponential function, and proposed different functions for certain bridge structural systems.

Clauwaert and De Backer (1982) presented a mathematical model based on a two-degree-of-freedom dynamic system developed to estimate the dynamic loading conditions of bridge expansion joints. The interactions between axle loads and roadway profile surrounding expansion joints were also investigated. A two-axle vehicle was used to validate the mathematical model and good agreement was found between the calculated and observed values of dynamic amplification.

Billing (1982, 1984) reported the experimental findings of one of the most important full-scale dynamic testing programs on highway bridges. The tests were conducted on 27 bridges of different types in 1980 and formed the third series of tests undertaken for the Ministry of Transportation of Ontario (MTO). The results served in the elaboration of the Ontario Highway Bridge Design Code (Billing and Green 1984; OHBDC 1983*a*, 1983*b*). These results will be discussed in more detail in the following sections.

Harmann *et al.* (1984) presented statistical methods of evaluating traffic loading effects on bridges, using the data from traffic surveys. These methods were aimed at the calibration of statistically based design codes. The authors also compared their methods with the specifications of AASHTO and the ASCE Committee on Loads and Forces on Bridges (ASCE 1981).

Clauwaert (1984*a*) presented two methods for the evaluation of dynamic forces induced by a single-axle vehicle on a roadway profile. Experimental tests were also carried out to validate these methods. This study on roadway profile and vehicle interaction was part of a large Belgian research program on the mechanical behaviour of bridge expansion joints (see also Clauwaert and De Backer (1982) reviewed above). Two algorithms were developed: one for the evaluation of the response induced by any kind of obstacles, and one using Fourier spectra of roadway profiles to determine the dynamic response of the bridge.

Gates and Smith (1984) reported the experimental results from ambient vibration testings of 57 typical bridges located in California, U.S.A. Seismometers, placed at various locations on the structures, were used to measure the velocity data under normal traffic and wind loadings. The vibration frequencies and the mode shapes were determined from the measured data and compared with the finite element models which were generally satisfactory. The authors also showed that care had to be taken when modelling boundary conditions at the supports and section properties.

Palamas *et al.* (1985) presented a theoretical study of the effects of surface irregularities on the dynamic response of bridges under suspended moving loads. The paper dealt with geometrical imperfections of two types: (*i*) global, simulating irregularities due to permanent loads, creep or prestressed forces; and (*ii*) local, representing joints or local defects. A single-degree-of-freedom system was used for the vehicle and a Rayleigh-Ritz method was used for the dynamic analysis. This study showed that in some cases, the DAF could be two to three times that recommended by current international design codes, suggesting that irregularities could no longer be neglected. It also showed that speed limitations did not always guarantee a decrease in dynamic effects.

Clauwaert (1986) presented the method used by the Centre de Recherches Routières de Belgique for the determination of vertical loading on bridge joints. The loading resulted from a conventional static force and a fatigue-related force. A formula giving the deterioration state of the joint was suggested, and accounted for the age of the bridge, the frequency of commercial vehicles, the yearly rate of increase in commercial vehicles, and the number of axles. This paper was also part of a Belgian research program on bridge expansion joints (Clauwaert 1984*b*).

Wilson (1986) examined the results from the recorded data on twelve strong-motion accelerograms which were placed on a highway bridge subjected to the August 6, 1979, Coyote Lake, California, earthquake. Optimal modal parameters for linear-models were determined to replicate the observed time-domain seismic response of the structure. As the intensity of shaking increased, the frequency of each mode generally decreased while the fundamental mode damping increased. A full three-dimensional model was developed and the computed response was compared with the measured data. The finite element model could predict the first two frequencies accurately only when the expansion joints were assumed to be locked. This phenomenon was also observed in the deformations of the structure in the fundamental mode.

Kato and Shimada (1986) conducted dynamic tests on an existing prestressed concrete bridge during its failure test. The static load was applied at the centre of the span by two hydraulic jacks. The dynamic testing had to be carried out simultaneously and was therefore based on an ambient vibration method. In this way, the change in the dynamic properties of the bridge due to its deterioration during the failure test was monitored. In particular, it was found that the natural frequency substantially and rapidly decreased as the load approached the ultimate load, whereas little change in the damping values was observed.

Cantieni (1983, 1984*a*, 1987) presented the results of dynamic experimental investigations on Swiss highway bridges. These studies were conducted under the auspices of the Federal Laboratory for Testing of Materials (EMPA). More than 200 highway bridges were tested over many years and the conclusions achieved in these studies allowed for new recommendations on the dynamic amplification factor to be inserted in the new Swiss SIA 160:1988 in revision of the SIA 160:1972 standard. Thus, SIA 160:1988 was the second code, after that of Ontario (OHBDC 1979), that not only recognized the importance of the natural frequency of the bridge as did ASCE (ASCE 1980), but also introduced a dynamic amplification factor in terms of natural frequency. The experimental findings of the EMPA are discussed in the following section.

Inbanathan and Wieland (1987) presented an analytical investigation on the dynamic response of a simply supported box girder bridge due to a moving vehicle. In particular, they considered the profile of the roadway using a response spectrum and 10 artificially generated time history loads for speeds of 19 and 38 km/h. The study of the response of a bridge due to a generated dynamic force was justified in view of the random nature of the problem. Some of the findings reported were the following:

1. The effect of vehicle mass on the bridge response is more significant for high speeds.
2. The maximum response is not affected by damping.
3. The stresses developed by a heavy vehicle moving over a rough surface at high speeds exceed those recommended by current bridge design codes.

Flesch and Kernbichler (1987) presented a dynamic method for the damage evaluation of large bridges. The method combines field tests and dynamic analysis. A model of the structure in its initial state is first established and compared with the results of in-situ tests conducted at different times. Long-term damage in a given structure will result in changes in its dynamic properties. The resulting patterns of change can be used for identification of damages. In view of the growing interest in long-term inspection for existing bridges when it is not possible to obtain initial models, a new method was presented, using structural dynamic modification (SDM) software. It was found that this method is a powerful means of estimating the sensitivity of modal parameters to changes in stiffness and damping.

Lee *et al.* (1987) reported the static and dynamic tests on an old reinforced concrete bridge before its demolition. The static test helped to calibrate a mathematical model and evaluate bearing restraint moments at the supports. The mathematical model was then used to compute the vibration frequencies of the structure. These frequencies were compared with the measured values obtained during dynamic tests carried out with a cylindrical steel hammer and a light truck. It was found that the dynamic modulus of elasticity was higher under dynamic load and produced results that were closer to field test results. The calculated vibration frequencies were higher when restraint effects were considered. The importance of carefully modelling boundary conditions, particularly restraint moments, was also demonstrated.

Tests have also been reported in Belgium by the Centre de Recherches Routières (Ververka and De Backer 1988) and by the Ministry of Public Works in Liège (Stifkens and Demars 1981; De Backer *et al.* 1981). The latter conducted extensive tests on the 6000 bridges that constitute the Belgian patrimony. These tests were designed for the determination of initial dynamic parameters of each separate bridge for long-term evaluation of successive degradations.

Coussy *et al.* (1989) presented a theoreticals study of the effects of random surface irregularities on the dynamic response of bridges under suspended moving loads. A single-degree-of-freedom oscillator was used for each axle of the vehicle, while the bridge was represented by an elastic beam with constant flexural rigidity and linearly distributed mass. The profile of the roadway was described by a random process with spectral density taken from previously reported experimental data. The authors concluded that the DAF decreases with span length but not as strongly as given by the codes of different countries. This was explained by the coupling of bridge and vehicle motion. The results also suggest that the DAF is independent of the span length in the absence of surface irregularities.

Bakht and Pinjarkar (1989) presented a lucid comparison of different methods of calculating the DAF from test data. The authors showed that the axiomatic definition of eq. [2] has been interpreted in at least eight different ways by researchers throughout the world, making it difficult to compare results from field studies. Some of these formulas are reviewed in a following section. Among the factors that can lead to differences in the computed values of the DAF are the following:

1. Vehicle type: A "final" value of the DAF for a specific bridge should be obtained from repeated dynamic testings under normal traffic conditions.
2. Vehicle weight: It has been shown that the DAF decreases with an increase of the vehicle weight. Therefore, the data obtained from lightly loaded vehicles will lead to an overestimation of the DAF and should not be considered when computing this factor.
3. Vehicle position: It has been observed that the DAF obtained from measurement points far from the vehicle position is higher than that for a measurement point located directly under the vehicle load. The data obtained from such measurement points should not be used to compute the DAF (see the following section).
4. Type of measurements: The DAF is usually computed from vertical deflection measurements. Strain measurements have also been used and it has been observed that the DAF values obtained from such measurements are usually smaller. Without bearing restraint, the girder strains are proportional to the girder moments, and can be used directly to compute the DAF for girder moments. However, it is still unclear whether the DAF values obtained from deflections or strain measurements can be applied directly to other responses.
5. Bearing restraint effects: In the presence of bearing restraint forces, girder strains are no longer proportional to girder moments and the DAF values obtained from strain measurements do not represent moment amplification. These forces also stiffen the bridge and can explain differences between the calculated and measured frequencies.

The authors concluded that the DAF is not a deterministic quantity and should be evaluated by a probabilistic means and recommended a procedure for obtaining the DAF from dynamic tests.

Eymard *et al.* (1990) presented an analytical model for the dynamic behaviour of bridges under full traffic. A simple elastic beam was used for the bridge and a single-axle vehicle was represented by a damped suspended mass. The tires were modelled by a viscoelastic spring and the roadway roughness was described by a statistical function and was either measured or generated from a given spectral density. The friction system could be blocked either in compression or in tension. The authors concluded that the single mode dynamic model could successfully calculate the dynamic behaviour of bridges under real traffic, give the frequency histograms of the response, and evaluate lifetimes of various bridge elements.

Chan and O'Connor (1990*a*) presented a simple vehicle model in which each axle load consists of a constant component and a sinusoidally varying dynamic component. The amplitude of the dynamic component was chosen equal to 10% of the static axle load. This upper-bound value was calibrated by field studies (Chan and O'Connor 1990*b*) where wheel loads were obtained from measured strains on a highway bridge, and by test data taken from Cantieni (1983) and other sources. An alternative way of expressing dynamic amplification was proposed: the dynamic moment ratio DMR — taken as the ratio of maximum dynamic moment to the product of the span length, L, and the vehicle gross weight, ΣW_i (where W_i is the load for axle i) — noting that for a large-wheel-base vehicle, the maximum static response will be small and this will lead to an overestimation of the DAF as defined by eq. [2]. The maximum bending moments for a large number of simply supported bridges and different axle configurations were computed, using a simplified beam model for the bridges and introducing a bridge factor, BF = Lf_0/v, relating the first flexural frequency, f_0, to the span length, L, and the vehicle speed, v. The following conclusions were made:

1. The maximum dynamic amplification occurs when the dynamic component of the load varies at the bridge's first flexural frequency, leading to high values of the DAF.
2. Large dynamic amplification occurs when the maximum response due to succeeding axle loads coincide.
3. For high dynamic amplifications, the axles within a group (tandem or tri-axle) can be replaced by a single-axle load at the centre of the group.
4. As the vehicle weight increases, although the maximum response increases, the DAF decreases and the DMR reduced even more significantly.
5. Computed and measured amplification factors were found in excess of code values.

Billing and Agarwal (1990) recommended standard methods that might be used for dynamic testings of highway bridges, underlining the fact that although many test data were available in the literature, the methodology used for these tests was often unclear and the procedures used for data processing were not specified. They addressed the following issues, based on the experience acquired during a program of tests conducted by the Ontario Ministry of Transportation and Communications (Billing 1982):

1. Logistics of bridge testing: bridge selection, traffic control, equipment location, test team, protection against theft or vandalism, etc.
2. Safety: collaboration with the police, weather conditions, reliable radio communications, etc.
3. Instrumentation: minimum number of instruments needed, installation and wiring of strain gauges, displacement transducers, accelerometers and trigger devices (such as pressure tubes).
4. Data capture: type of data acquisition system, calibration, etc.
5. Test procedure: number, location, and speed of test vehicles, sequence of events in a typical test, use of normal uncontrolled traffic.
6. Data processing: calibration, trend removal, filtering, and modal analysis.
7. Data analysis: computing of the DAF (see following sections) and other bridge responses.

These recommendations, aimed at establishing testing procedures, are at the core of this research report which is part of a study whose ultimate goal is to develop such standard methods for the experimental evaluation of the DAF.

Agarwal and Billing (1990) reported the dynamic testing of a bridge exhibiting a high dynamic response and display-

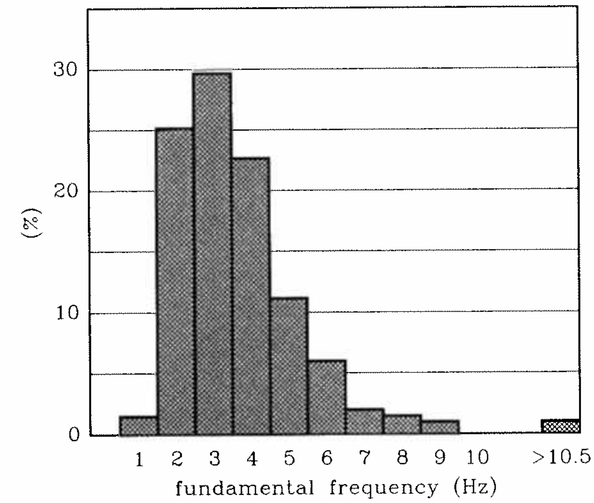

FIG. 2. Distribution of fundamental natural frequency of bridges for 202 values: min = 1.23 Hz; mean = 3.66 Hz; max = 14 Hz (Cantieni 1983).

ing complex vibrations in torsional and flexural modes. The first four modes of the bridge were found to be in the 2–5 Hz range, corresponding to the resonant frequencies of commercial traffic suspension systems. The dynamic amplification was lower for side-by-side vehicle and could be much larger for multiple presence of vehicles on one lane and for multi-lane loading with staggered vehicles. The authors also discussed a method for evaluating the DAF in continuous bridges, which would be more appropriate for fatigue consideration (reviewed in a following section).

Proulx and Paultre (1991) presented the results of the dynamic testing of a composite arch bridge. Values of the DAF were evaluated from controlled and normal traffic tests. They reported good agreement between measured frequencies and frequencies computed by finite element analysis. Influence of filtering techniques on the evaluation of the DAF was also discussed. Damping of the bridge was evaluated through free-vibration tests.

Hwang and Nowak (1991) presented a procedure to calculate statistical parameters for dynamic loading of bridges, to be used in design codes. These parameters, based on surveys and tests, included vehicle mass, suspension system and tires, and roadway roughness, which was simulated by stochastic processes (PSDs). The authors used a Monte Carlo method to represent the random nature of traffic, with variable vehicle characteristics, and studied the effects of the statistic parameters on dynamic loading. This procedure was applied to steel and prestressed concrete girder bridges, for single and side-by-side vehicle configurations. Values of the DAF were computed using prismatic beam models for the bridges and step-by-step integrations. It was found that (i) the DAF decreases with an increase in vehicle weight; (ii) the DAF for two side-by-side vehicles is lower than that for a single vehicle; and (iii) the dynamic load is generally uncorrelated with the static live load.

Dynamic amplification factors — a review

The most comprehensive full-scale dynamic testing programs were conducted in Switzerland (Cantieni 1983) and in Ontario (Wright and Green 1963; Csagoly *et al.* 1977; Billing 1982, 1984). In Switzerland, where the bridge design

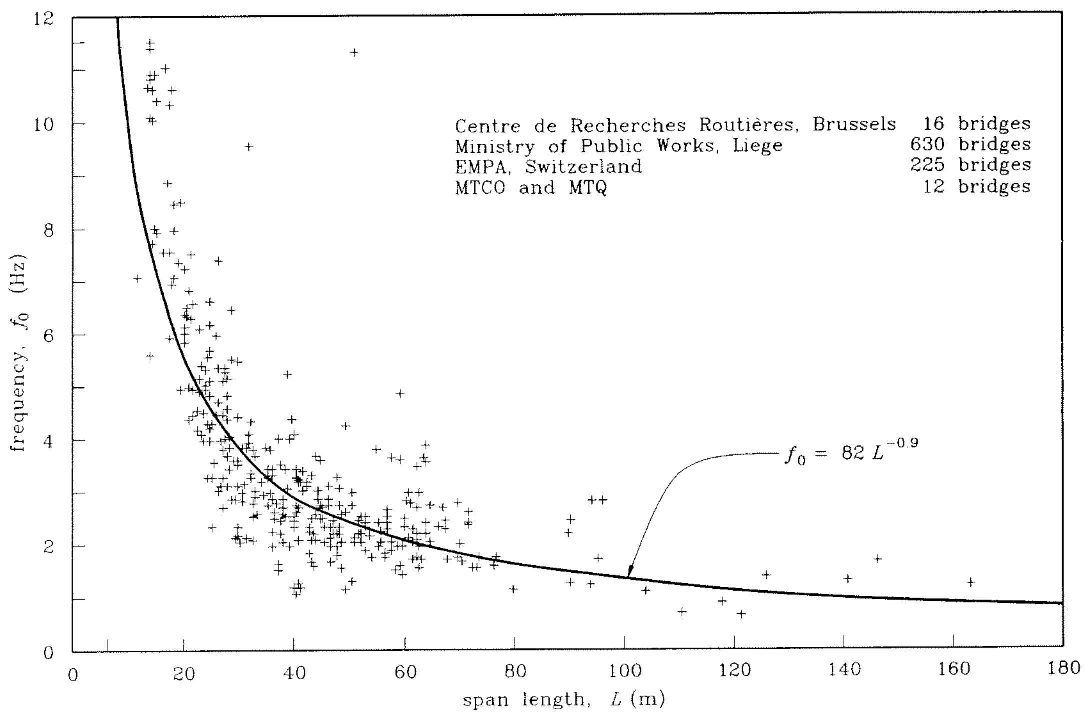

FIG. 3. Fundamental frequencies versus span length for 898 highway bridges.

standards specify that loading tests must be conducted on every bridge with a span exceeding 20 m, a considerable database of more than 200 bridges is now available. In Ontario, 52 bridges were tested in 1956–1957, 11 bridges in 1972, and, more recently, 27 bridges in 1980 and 1981 to provide information supporting design code provisions. In these reports, many parameters influencing the dynamic response of bridges are considered. In particular, the DAF is now related to the bridge's fundamental frequency rather than to its span length. Even though the first flexural mode may not dominate the response, it is more convenient in design code specifications to relate the dynamic load allowance to this quantity. Other parameters, including roadway roughness, bridge damping, bridge geometry, and vehicle characteristics, are difficult to account for in code format. However, their effect should be considered in the evaluation of the DAF for a specific bridge. A review of the experimental findings on these parameters is presented in the following sections.

Fundamental frequency of the bridge

The natural vibration frequency of a bridge has a considerable influence on its dynamic response. It is now well established that the majority of modern highway bridges have fundamental frequencies in the range of 2–5 Hz, corresponding to the resonant frequencies of commercial vehicles (see Fig. 2). This frequency matching can lead to an important amplification of the dynamic response. Good agreement was found between vibration frequencies evaluated by computer modelling and those obtained by dynamic testing. Relations between bridge span and fundamental frequencies have been proposed. A typical relation is displayed in Fig. 3.

Roadway roughness

In the majority of field tests, roadway imperfections and irregularities were found to be a major factor influencing bridge response. This phenomenon is clearly illustrated in Fig. 4, representing the dynamic amplification factor on smooth and deformed roadway surfaces, where 50 mm thick wooden planks were used to simulate roadway roughness (Cantieni 1983).

The experimental findings seem to indicate that the most important impact forces are likely to occur at the bridge approach (see Fig. 5) as the vehicle crosses the joint irregularities that exist between the bridge and the abutments (Tilly 1985).

It was also observed that the amplification effect produced by roadway irregularities increases as the bridge stiffness increases (see Figs. 6 and 7 (Cantieni 1983).

Damping characteristics of the bridge

Damping values for 225 bridges located in Europe were obtained in various field tests and are summarized in Table 1, along with damping values for 19 out of 27 bridges recently tested in Ontario (Billing 1984).

Note that the damping values obtained by Billing are relatively small compared with the damping values reported by Tilly. This is probably due to the use of different methods for the evaluation of damping (logarithmic decrement, half-power bandwidth, etc.). Although high levels of damping reduce dynamic response, more research is required before the exact influence of damping on the DAF can be ascertained. There does not appear to be sufficient information to claim that a weakly damped bridge has a relatively high dynamic amplification.

Excitation source

Many sources of dynamic excitation have been used to evaluate the dynamic characteristics of bridges. Some of the methods used to determine vibration frequencies, mode shapes, and damping are (*i*) wind-induced vibrations, efficient only for flexible bridges, such as suspension and cable-

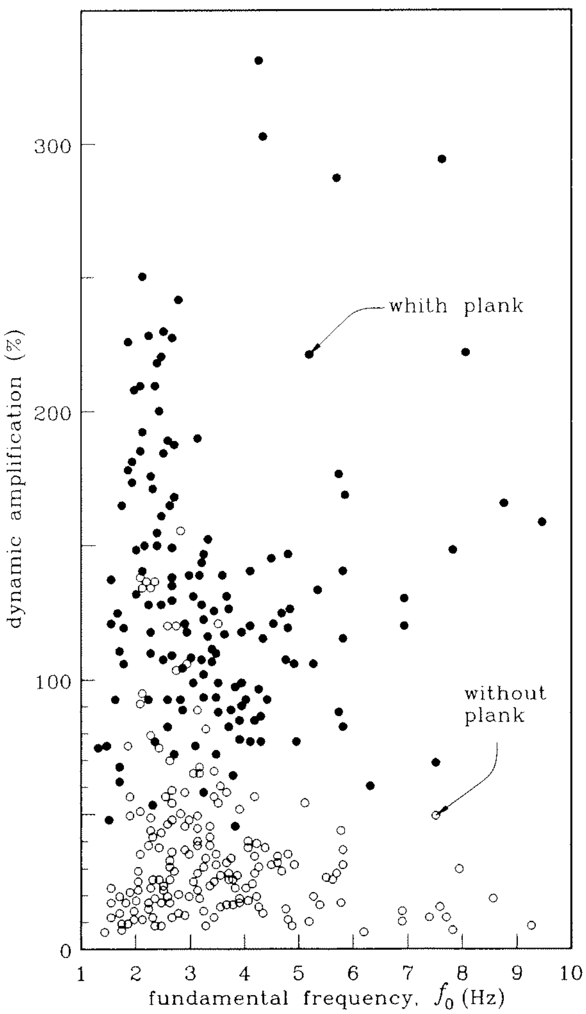

FIG. 4. Effect of roadway roughness simulated by a plank on the maximum dynamic amplification (adapted from Cantieni, 1983): (*i*) with a plank: min = 43%; mean = 130%; max = 450%; (*ii*) without a plank: min = 2%; mean = 30%; max = 102%.

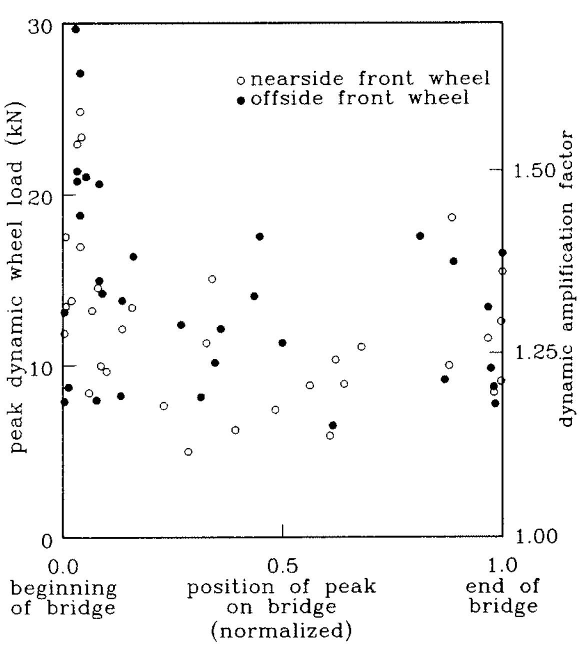

FIG. 5. Position on the bridge where the peak amplification occurs at vehicle passage (adapted from Tilly, 1986).

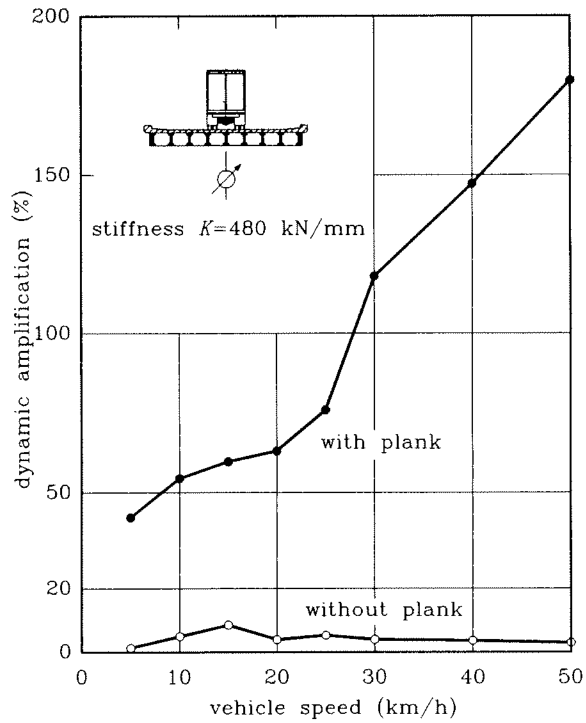

FIG. 6. Amplification effect produced by roadway irregularities increases with the bridge stiffness — stiff bridge (adapted from Cantieni, 1983).

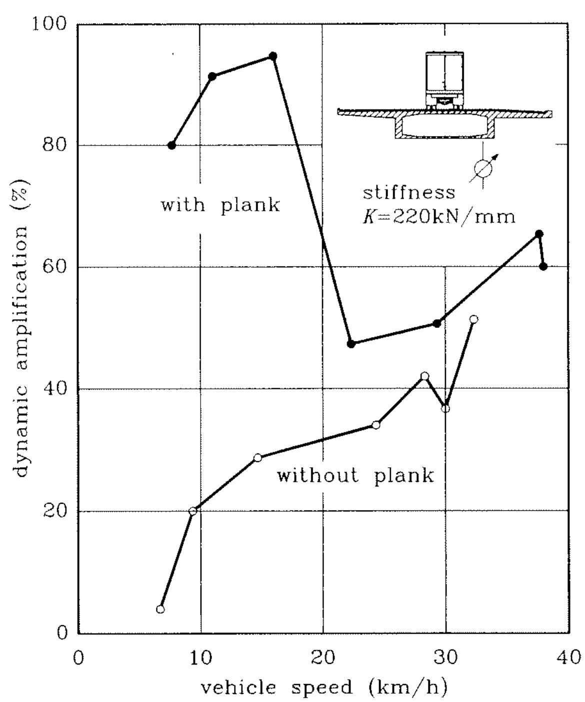

FIG. 7. Amplification effect produced by roadway irregularities increases with the bridge stiffness — flexible bridge (adapted from Cantieni, 1983).

stayed bridges; (*ii*) eccentric mass shakers and servo-hydraulic reactional mass exciters, allowing the excitation of the stucture at predetermined frequencies; (*iii*) sudden release of static load or imposed displacement; and (*iv*) instrumented hammer used in modal testing methods.

Only dynamic testing under traffic loading, however, is

TABLE 1. Typical values of measured damping of highway bridges

Type of bridge	Span length (m)	Number of bridges tested	Average damping value	Lowest damping measured
Concrete in Switzerland, Great Britain, and Belgium (Tilly 1986)	10–85	213	0.079	0.020
Composite, steel–concrete in Great Britain (Tilly 1986)	28–41	12	0.084	0.055
Prestressed concrete (Billing 1984)	8–42	4	0.022	0.008
Steel (Billing 1984)	4–122	14	0.013	0.004

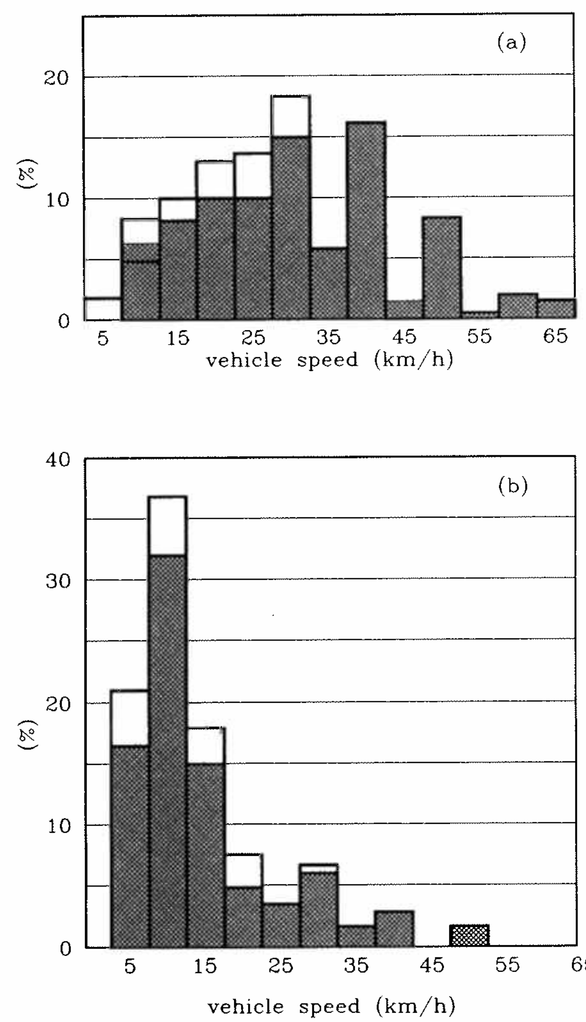

FIG. 8. The effect of roadway roughness on the value of the critical speed at which the largest dynamic amplification was measured (adapted from Cantieni, 1983): (*a*) without a plank (208 values); min = 5 km/h; mean = 29 km/h; max = 66 km/h (shaded area: 180 tests in which the maximum attained speed was greater than 30 km/h); (*b*) with a plank (203 values); min = 3 km/h; mean = 15 km/h; max = 50 km/h (shaded area: 175 tests in which the maximum attained speed was greater than 30 km/h).

suitable for the evaluation of the DAF. If test vehicles are used, vehicle speed, weight, axle spacing, and position on the bridge can be controlled. Dynamic wheel load can also be obtained from tire pressures (Cantieni 1983) and pavement irregularities can be evaluated (profilometers, spectral density techniques). It must be recognized that this is not representative of the actual conditions of loading of highway bridges. It is therefore logical to think that the DAF can be reliably estimated only under normal uncontrolled traffic loading (Bahkt and Pinjarkar 1989). This method is also the only way to enable comparisons with the existing test results as well as between different bridges and different instantaneous behaviours of the same bridge in relation to long-term routine inspections.

Vehicle speed

It was found that heavy commercial vehicles exhibit two vibration modes: body-bounce vibrations at frequencies ranging from 2 to 5 Hz and wheel-hop vibrations at frequencies greater than 7 Hz. Roadway roughness and vehicle speed, which can privilege one particular mode, govern these vibrations (Cantieni 1983). The influence of vehicle speed on the DAF can be seen from the critical speed distribution displayed in Fig. 8 corresponding to tests with and without planks. The critical speeds for the case with a plank are clearly concentrated in the 5–15 km/h range.

Bridge geometry

Figure 9 illustrates the effects of bridge geometry on dynamic amplification, distinguishing between beam-type bridges and bridges having particular geometries such as horizontal curvature, skewed deck, skewed supports, etc. The results reported by Cantieni (1983) show a slight tendency that straight beam-type bridges display higher dynamic increments.

The dynamic behaviour of skewed or extremely curved bridges are characterized by the presence of closely spaced flexural and torsional modes. If more than one of these modes lie in an excitation range of the test vehicle, it becomes difficult to attribute the response to a particular mode (Cantieni 1983).

Construction materials

The type of construction material does not seem to play an influencing role on the DAF. The experimental findings do not show any particular tendency for a specific construction scheme.

Test vehicles

The influence of mechanical properties of test vehicles on the dynamic response is illustrated in Fig. 10 representing the tire rigidity and the excitation amplitude effects.

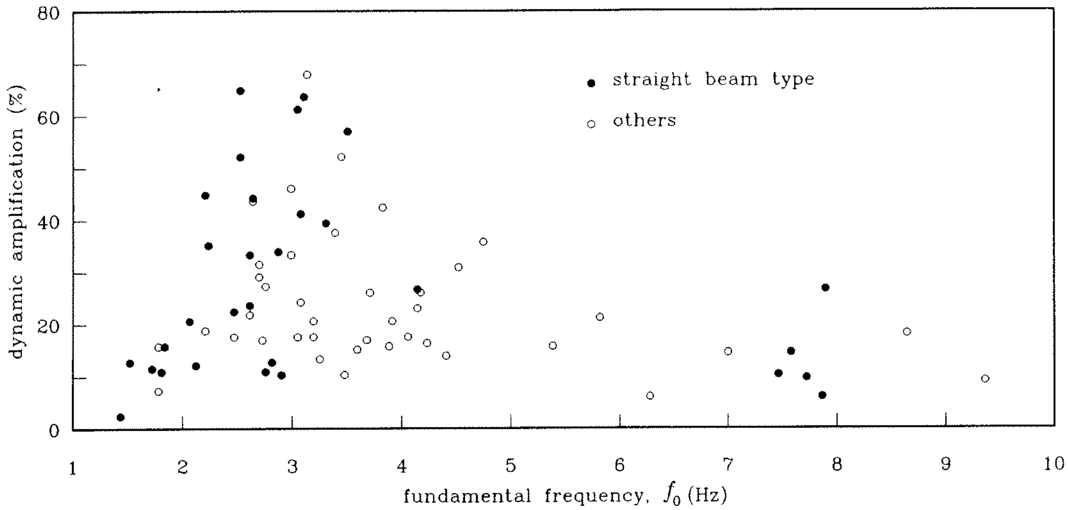

FIG. 9. Effect of bridge geometry on the dynamic amplification (adapted from Cantieni, 1983).

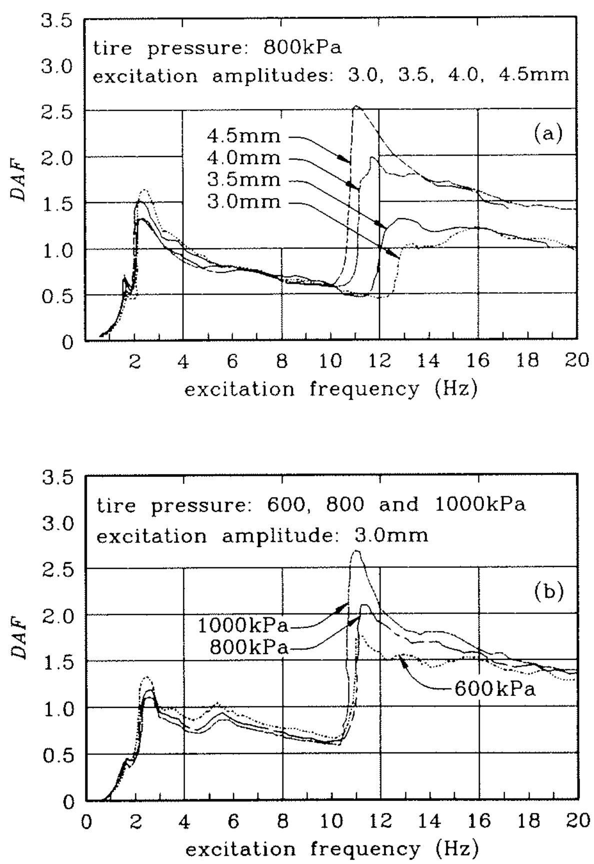

FIG. 10. Influence of mechanical properties of test vehicles on the DAF (Tilly 1986): (*a*) effect of excitation amplitude; (*b*) effect of tire pressure.

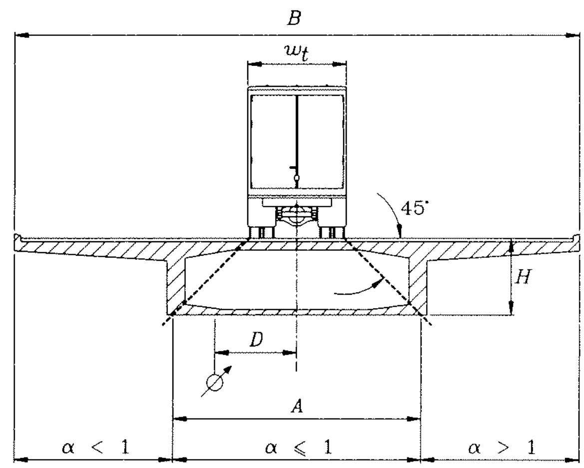

FIG. 11. Definition of the transverse influence zone in which the static response is maximum (adapted from Cantieni, 1984).

Very stiff bridges, with fundamental frequencies in the range of 10–15 Hz, are more influenced by vehicle mechanical properties than most modern highway bridges, with frequencies lying in the range of 2–5 Hz.

Position of the measurement points and number of instruments used

A sufficient number of instruments is requied to determine the vibration mode shapes and the deflection patterns. The positioning of these instruments should be carefully chosen from the vibration mode shapes obtained by computer modelling of the bridge under consideration. For a given section, instruments lying outside an influence region can lead to excessive values of dynamic amplification, caused by the small static response measured at these points. Cantieni (1983) defines this influence zone with the α parameter defined as

$$[4] \quad a = \frac{D}{H + 0.5w_t}$$

where D is the distance between the measurement point and the driving axis, H is the height of the cross section at the measurement point, and w_t is the truck width (see Fig. 11).

If $\alpha \leq 1.0$, the measurement point lies within the direct influence region of the vehicle, and if $\alpha > 1.0$, the value of the DAF will be overestimated as the static response, R_{sta}, will be smaller for this point. Values of dynamic amplification as high as 400% have been observed in some cases such as the flexible bridge transverse section illustrated in Fig. 12, where transverse displacement distribution is not uniform (see top of Fig. 12). Therefore, only the response quantities obtained from measurement point 3 can be used

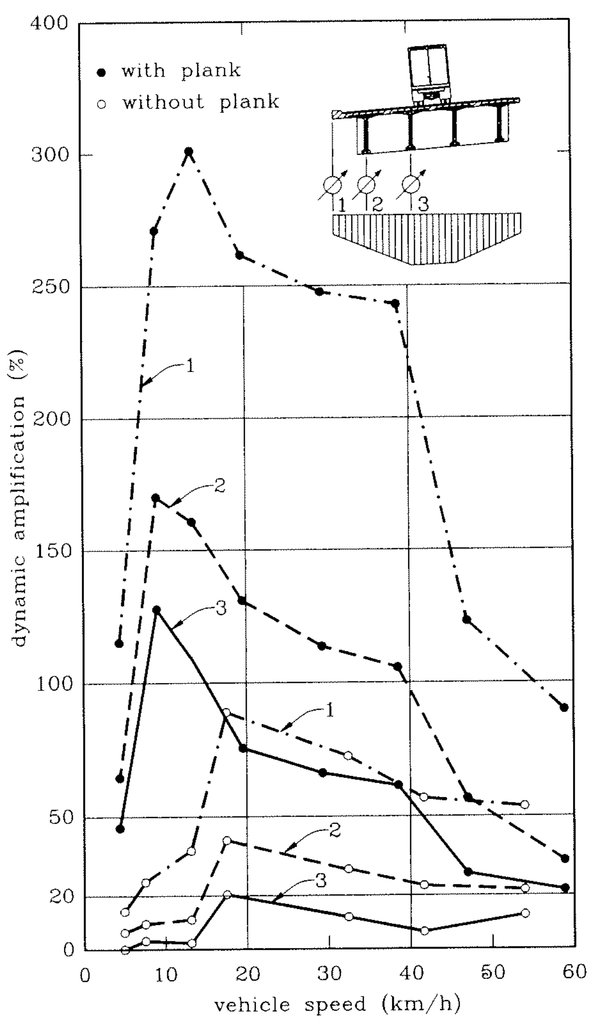

FIG. 12. Influence of transverse section stiffness on the dynamic amplification evaluation for a bridge having a flexible transverse section. Only the results obtained from measurement point 3 are considered for the DAF (adapted from Cantieni, 1983).

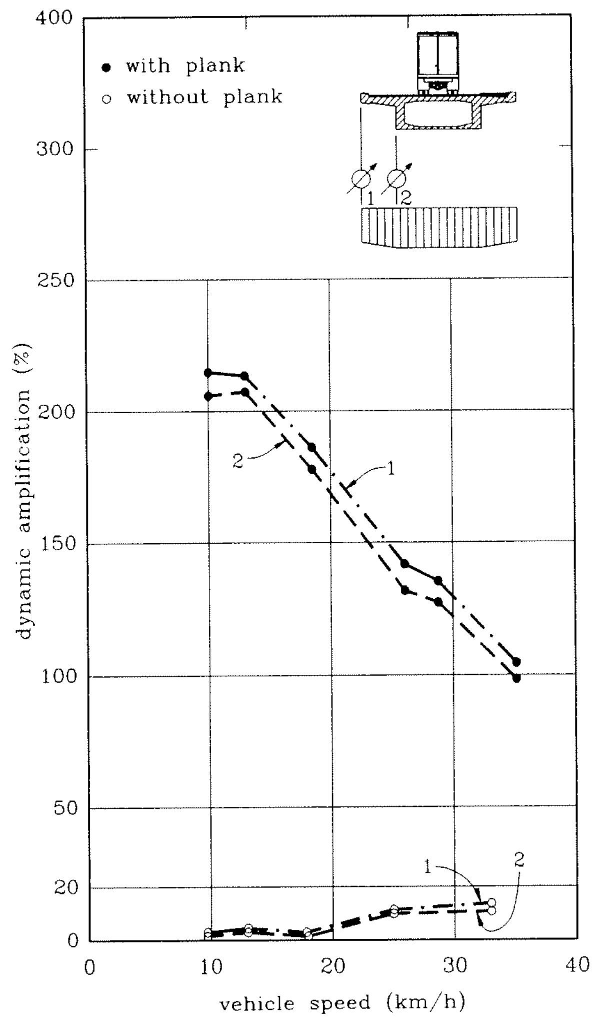

FIG. 13. Influence of transverse section stiffness on the dynamic amplification evaluation for a bridge having a stiff tranverse section. The results obtained from all measurement points are considered for the DAF (adapted from Cantieni, 1983).

for the evaluation of the DAF. For the stiff bridge transverse section illustrated in Fig. 13, the results obtained at all measurement points can be considered accurate, as the transverse displacement is almost constant over the entire cross section (see top of Fig. 13).

Wheel dynamic loading measurement

Although knowledge of the dynamic wheel loading is not necessary for evaluation of the DAF, its importance has been pointed out in the literature. When shakers or exciters are used, the excitation force is known, and the true frequency response functions (FRFs) can be computed for the evaluation of vibration frequencies and mode shapes. When testing is done under traffic loading, this force is not usually measured, and the frequency response functions are computed for each instrument with respect to a selected reference instrument. This technique has been successfully used in Ontario (Billing 1982) to compute bridge dynamic properties, but a measurement of dynamic wheel loads can lead to a better understanding of bridge–vehicle interaction. Cantieni (1983) reported experimental procedures for the evaluation of dynamic loading, in which wheel loads were recorded simultaneously with the bridge response, based on tire deflection.

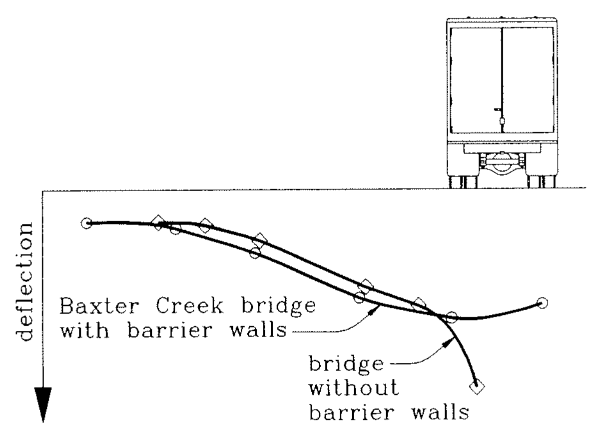

FIG. 14. Stiffening effect of barrier walls (adapted from Billing, 1984).

Secondary elements

It has been found that, in some cases, the contribution of secondary elements, such as footways, parapets, and bracing systems, to the stiffness of the bridge cannot be neglected. This is illustrated in Fig. 14 (Billing 1984) where

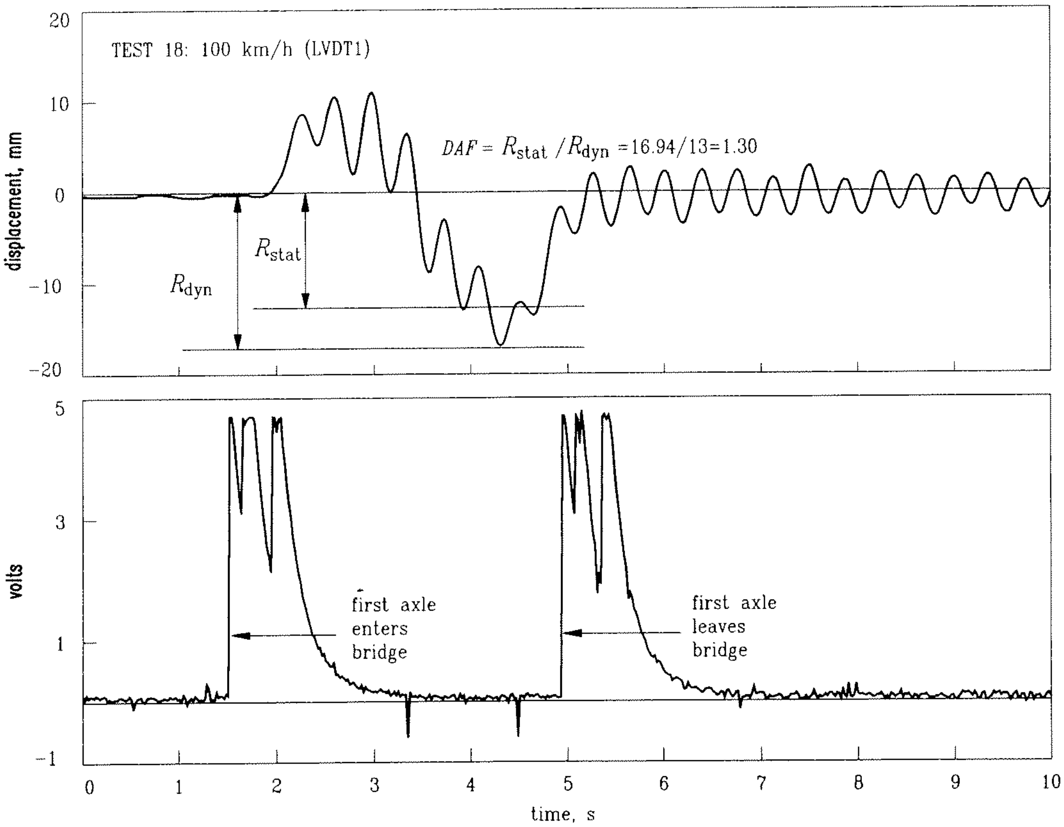

FIG. 15. Evaluation of the DAF from displacement time-history.

the influence of secondary elements (barrier walls, curbs, etc.) on the transverse static response distribution is shown. This figure compares the mid-span transverse deflections of two similar three-span continuous bridges, one with barrier walls (Baxter Creek Bridge) and one without barrier walls.

Measured quantity

The choice of appropriate instruments depends on the type of response to be measured. For the evaluation of the DAF, displacement transducers yielding vertical displacement time-histories and strain gauges have been used, although it has been found that strain gauges could lead to smaller values of the DAF. For the evaluation of vibration frequencies and mode shapes, accelerometers or seismometers have been used. Some researchers have obtained displacements by integrating acceleration response, but this method is not practical and there is a loss of accuracy in the operation. Cantieni (1984) also reported a sophisticated laser system for displacement measurement.

Computing the DAF from recorded data

In their reviews of dynamic testing, Bakht and Pinjarkar (1989) compiled eight different formulas for computing the DAF from recorded data. Among these formulas, four are still used by researchers. The first one is given by

$$[5] \quad \mathrm{DAF} = \frac{R_{\mathrm{dyn}}}{R_{\mathrm{fil}}^{\mathrm{dyn}}}$$

where R_{dyn} is the maximum dynamic response and $R_{\mathrm{fil}}^{\mathrm{dyn}}$ is the static response obtained by filtering the dynamic response, taken at the time when the maximum dynamic response occurs. Using the static response obtained from tests at crawling speed, $R_{\mathrm{sta}}^{\mathrm{dyn}}$, we have

$$[6] \quad \mathrm{DAF} = \frac{R_{\mathrm{dyn}}}{R_{\mathrm{sta}}^{\mathrm{dyn}}}$$

Using the maximum static response obtained by filtering the dynamic response, R_{fil}, we have

$$[7] \quad \mathrm{DAF} = \frac{R_{\mathrm{dyn}}}{R_{\mathrm{fil}}}$$

or, using the maximum static response obtained from tests at crawling speed, R_{sta},

$$[8] \quad \mathrm{DAF} = \frac{R_{\mathrm{dyn}}}{R_{\mathrm{sta}}}$$

When the filtered response is close to the static response obtained from crawling speed tests, [5] and [6] are the same, as well as [7] and [8]. Equations [5] and [6] have been used in Switzerland (Cantieni 1983) and [7] and [8], which correspond to [2] have been used in Ontario (Billing 1982). Figure 15 illustrates the use of [8] for data recorded on the Milnikek Bridge (Proulx and Paultre 1991).

The maximum dynamic response is obtained directly by taking the maximum value of the measured data, and the reference value, or maximum static value, can be estimated by the following means:

1. By performing quasi-static tests where the test vehicles move at sufficiently low speed (5–15 km/h), or by taking several measurements for different positions of the vehicles at rest on the bridge — This method yields the true static

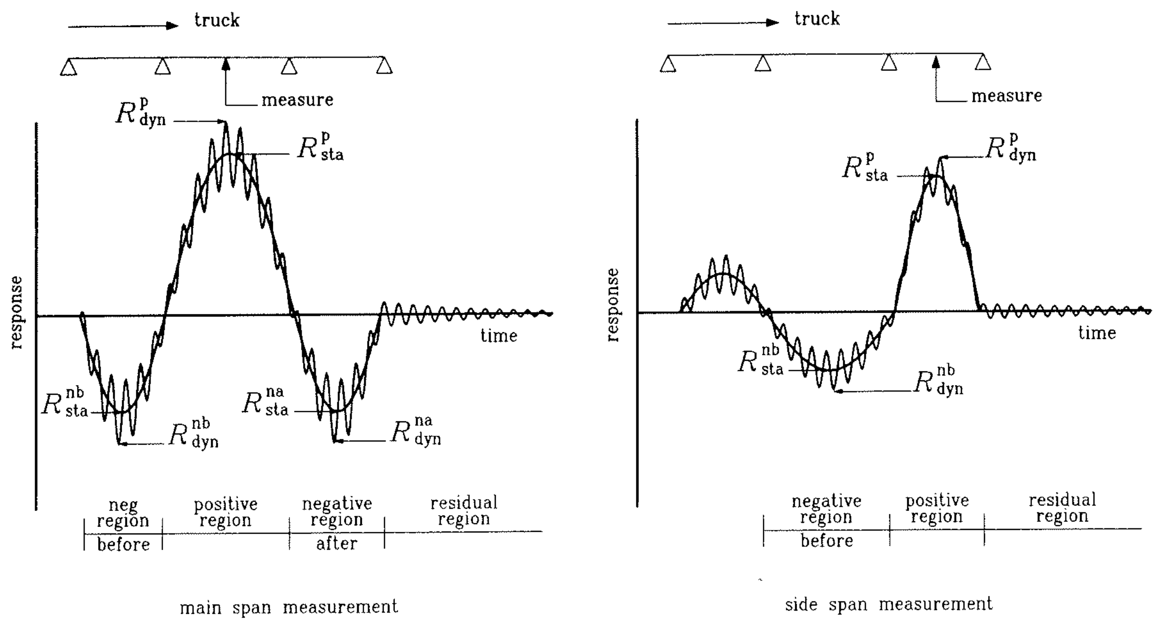

FIG. 16. Evaluation of the dynamic amplification factor for multispan bridge (adapted from Billing, 1982).

deflection or strain for each instrument and for a given test vehicle. It can be used for multiple as well as single vehicle tests, and may be applied in an ambient traffic test, where the characteristics of the vehicles are unknown. However, subsequent tests at high speed may not be carried out at the exact same speed or location, and may not generate the same static response. It should therefore be considered only as a good approximation of the static response for runs at high speed.

2. By filtering the time-history of the measured quantity — In this method, a lowpass digital filter, applied to the recorded data, is used to "smooth-out" the dynamic frequencies in the signal. The filtering can be done in the time domain with moving average, or finite-impulse response (FIR) filters. The filter parameters are chosen to eliminate the dynamic components of the signal. To achieve this, the filter must have a passband of v/L Hz, where v is the vehicle speed and L is the shortest span length, and a stopband with a cutoff frequency below the bridge's first fundamental frequency. The transient effects of the digital filter can be eliminated by using a long leader of recorded data before the test vehicle enters the bridge. The leader also prevents the filtered data from showing the filter's impulse response at the beginning of the record (Billing 1982). This method can be used in the same conditions as the quasi-static tests. However, there are some possible sources of errors: (*i*) for short span bridges (≤ 15 m), the static frequency, v/L, becomes equal to (and even higher than) the first vibration mode of the bridge, and the dynamic components of the signal can no longer be removed without affecting the static component, therefore limiting this technique to bridges with longer spans; and (*ii*) the compared DAF is dependent on the filter's "slope" in the passband; as the passband and stopband cutoff frequencies become closer, the value of the DAF increases, thus the need for comparison with the measured maximum static displacement obtained from the quasi-static tests.

3. Analytically, by finite-element analysis, where the static displacement or strain can be computed for a given test vehicle — This method is limited to controlled traffic tests as the weight and the exact position of the vehicle on the driving lane have to be known. It should therefore be used only to verify the results obtained from the two previous methods.

The evaluation of the DAF can be refined when considering a bridge having more than one span. Billing (1982) defines separate regions for a typical three-span bridge, and response instruments are placed at locations where maximum response is expected. Figure 16 shows three response regions for such a bridge.

In the positive region, the vehicle is located in the same span as the instrument; in the negative region, the vehicle is located in the adjacent span; and in the residual region, the bridge is in free vibration. Therefore a single span bridge would only have a positive region. The peak values shown in Fig. 16 are

1. R^{p}_{dyn}: dynamic response, positive region;
2. R^{p}_{sta}: static response, positive region;
3. R^{nb}_{dyn}: dynamic response, negative region, before the vehicle reaches the span where the instrument is located;
4. R^{nb}_{sta}: static response, negative region, before the vehicle reaches the span where the instrument is located;
5. R^{na}_{dyn}: dynamic response, negative region, after the vehicle leaves the span where the instrument is located;
6. R^{na}_{sta}: static response, negative region, after the vehicle leaves the span where the instrument is located; and
7. R^{r}_{dyn}: dynamic response, residual region.

Once these peak values have been obtained by filtering the signals, three dynamic amplification factors can be computed: DAF_{pos}, for the positive region; DAF_{neg}, for the negative region; and DAF_{res}, for the residual region:

$$[9] \quad \mathrm{DAF}_{pos} = 1 + \frac{(R^{p}_{dyn} - R^{p}_{sta})}{R^{px}_{sta}}$$

$$[10] \quad M = \max\{(R^{nb}_{dyn} - R^{nb}_{sta}),\ (R^{na}_{dyn} - R^{na}_{sta})\}$$

$$[11] \quad DAF_{neg} = 1 + \frac{M}{R_{sta}^{px}}$$

$$[12] \quad DAF_{res} = 1 + \frac{R_{dyn}^{r}}{R_{sta}^{px}}$$

where R_{sta}^{px} is the greatest static response for a group of instruments located on the same section across the driving lanes. In this way, the dynamic amplification factors are related to the largest positive region static response and can thus be directly compared. This ensures that the DAF is not overestimated. For example, Fig. 17 shows strain–time histories taken from four strain gauges located on the same section of the bridge, on girders 1 to 4.

Here, the largest static strain occurs on girder 3 and should be taken as a reference for equations [9], [11], and [12]. This may relate the dynamic response of one instrument to the static response of another, but it ensures that the largest dynamic response is related to the largest static response as required by current design codes. The different DAF values can then be averaged according to the lane of travel and the type, weight, and speed of the vehicle.

Agarwal and Billing (1990) discussed the use of a "response range" approach instead of the peak responses currently used for dynamic load allowance specifications. For a continuous bridge, the bridge components are subjected to a reversal of response as discussed above (Fig. 16). A DAF value defined for the response range could be defined as

$$[13] \quad DAF = 1 + \frac{(R_{dyn}^{n} + R_{dyn}^{p})}{(R_{sta}^{n} + R_{sta}^{p})}$$

where R_{dyn}^{n} and R_{sta}^{n} are the maximum dynamic and static responses in the negative regions, and R_{dyn}^{p} and R_{sta}^{p} are the maximum dynamic and static responses in the positive region. This approach leads to higher values of the DAF than those specified by design codes, but can be very significant for fatigue considerations in steel structures.

A procedure proposed in the Ontario code (OHBDC 1979) and reported by Bahkt and Pinjarkar (1989) recognizes the fact that the DAF is a nondeterministic quantity and should be treated on a statistical basis, as it is done for static live loads. This statistical value to be used in design, DAF_s, depends on the mean value of the dynamic amplification factor, DAF_{avg}, and the live load factor, α_L, specified by the Code:

$$[14] \quad DAF_s = \frac{DAF_{avg}\,(1 + c_v s \beta)}{\alpha_L}$$

where c_v is the coefficient of variation of the DAF, s is the separation factor for dynamic loading, and β is the safety index. A value of 0.57 has been found for the separation factor, and a value of 3.5 for the safety index is typically used for highway bridges.

Conclusions

In the evaluation of live loads to be used in bridge designs, two factors must be accounted for: the traffic loading and the dynamic load allowance. Traffic loading has received a lot of attention in the past and is well defined. Design codes represent the dynamic amplification phenomenon in a bridge by multiplying the static live loads by the dynamic load allowance. There has been a considerable amount of research conducted in the fields of bridge dynamics and, more recently, in the evaluation of dynamic amplification factors for existing bridges. A review of the analytical and experimental findings has suggested the following conclusions:

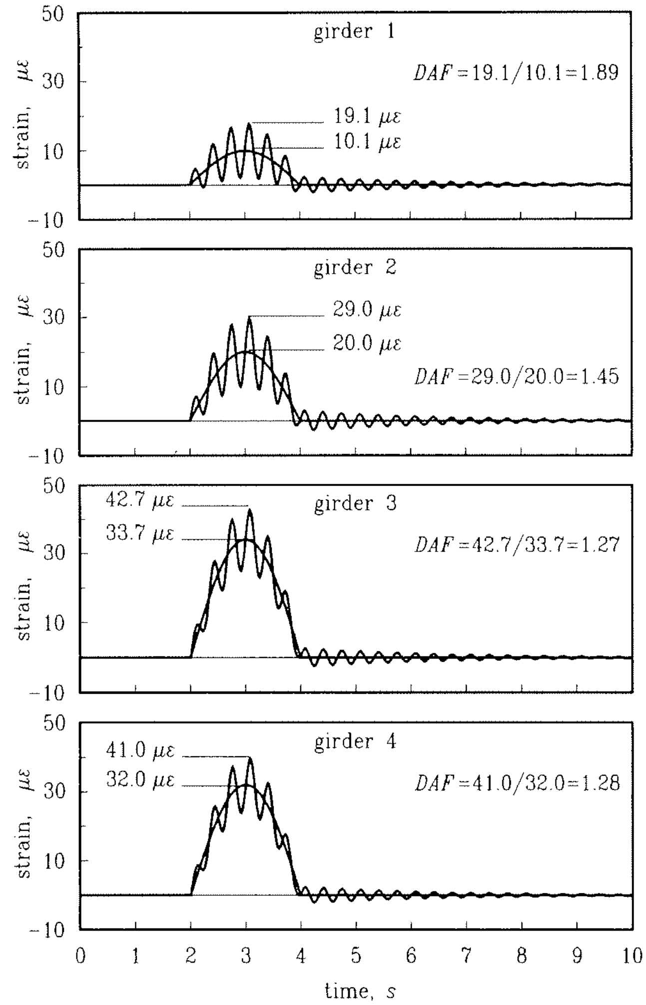

FIG. 17. Overestimation of dynamic amplification (adapted from Billing, 1982).

1. The DAF is related to the fundamental frequency of the bridge. The most significant traffic-induced vibrations are a combination of many modes, but it is easier for design code purposes to relate the dynamic load allowance to the fundamental frequency. Most modern highway bridges were found to have fundamental frequencies in the range of 2–5 Hz, corresponding to the resonant frequencies of commercial vehicles.

2. Even though specified live loads differ in most countries, this does not justify large differences in the dynamic load allowances for various national codes.

3. Analytical methods cannot reliably evaluate the DAF for a specific bridge, owing to the many parameters involved that are difficult to model. Moreover, such analyses would have to be carried out for a very large number of vehicles having different dynamic characteristics, making this approach unrealistic at the present time. However, computer

modelling was found to estimate vibration frequencies and mode shapes with good accuracy.

4. Full-scale testing under traffic loading is the only economical and practical way to evaluate the dynamic amplification factor with reasonable confidence. It is also a reliable method for determining structural dynamic properties and can be useful for inspection purposes.

5. Vehicle speed is an important parameter in the evaluation of the DAF. Dynamic testing under controlled traffic should be carried out over a sufficient range of speeds to cover critical speeds.

6. The peak value of the DAF is not strongly influenced by vehicle mass. Modifying the mass of the vehicle changes the velocity at which the maximum dynamic amplification occurs. However, for lightly loaded vehicles, the DAF value can be very large, due to small static response, and these vehicles should not be used for dynamic testing under traffic loading, except if the DAF is to be evaluated for such vehicles.

7. Roadway roughness and pavement irregularities strongly influence the DAF. Initial vibrations in the vehicle caused by uneven bridge approaches or expansion joints can lead to large values of the DAF.

8. Axle spacing of vehicles affects the amplitude of the maximum static response and hence the DAF.

9. Suspension systems of vehicles can have a significant influence on the DAF in the case of initial vibrations of these vehicles. Also, it was found that the DAF increases with tire pressure.

10. Bearing restraint forces influence vibration frequencies and should be considered in the prediction of the fundamental frequency.

11. More research is required before the exact influence of bridge damping on the DAF can be ascertained. There does not appear to be sufficient information to claim that a weakly damped bridge has a relatively high dynamic amplification.

12. Geometry and construction materials of typical highway bridges do not seem to play an influencing role on the value of the DAF.

13. It has been found that, in some case, the contribution of secondary elements such as footways, parapets, and bracing systems to the stiffness of the bridge cannot be neglected.

14. For dynamic testing, the position of measuring instruments on a transverse section can lead to excessive values of the DAF. Instruments that are outside an influence zone should not be considered when computing the DAF.

15. Many different formulas were suggested to evaluate the DAF from the experimental data obtained under traffic loading. More recently, researchers have used the ratio of maximum dynamic response over maximum filtered response as a definition of the DAF. A response range approach was also suggested for fatigue considerations.

16. Filtering techniques used to extract the static component from displacement or strain signals, obtained from dynamic testing under traffic loading, can have a considerable influence on the DAF. Proper filter characteristics should be used, and the static component should be compared with crawling speed tests and (or) static deflection predictions if available.

17. The DAF is a nondeterministic quantity and should be treated on a statistical basis.

This review clearly shows that there is a need to establish quantitative as well as qualitative test procedures and numerical signal processing standards based on international cooperation. This would result in a standardization of the experimental evaluation of the dynamic amplification factor.

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List of symbols

BF	bridge factor
c_v	coefficient of variation
D	distance between measurement point and driving axis
DA	dynamic amplification
DAF	dynamic amplification factor
$\mathrm{DAF}_{\mathrm{avg}}$	average dynamic amplification factor
$\mathrm{DAF}_{\mathrm{neg}}$	dynamic amplification factor, negative region
$\mathrm{DAF}_{\mathrm{pos}}$	dynamic amplification factor, positive region
$\mathrm{DAF}_{\mathrm{res}}$	dynamic amplification factor, residual region
$\mathrm{DAF}_{\mathrm{s}}$	statistical value of the dynamic amplification factor
DLA	dynamic load allowance
f_0	fundamental frequency of the bridge
L	bridge span length
$L_{\max}$	maximum bridge span length
H	height of bridge cross section
R_{dyn}	maximum dynamic response
$R_{\mathrm{dyn}}^{\mathrm{na}}$	dynamic response, negative region, after the vehicle leaves the span where the instrument is located
$R_{\mathrm{dyn}}^{\mathrm{nb}}$	dynamic response, negative region, before the vehicle reaches the span where the instrument is located
$R_{\mathrm{dyn}}^{\mathrm{p}}$	dynamic response, positive region
$R_{\mathrm{dyn}}^{\mathrm{r}}$	dynamic response, residual region
R_{sta}	maximum static response
$R_{\mathrm{sta}}^{\mathrm{dyn}}$	static response at time of maximum dynamic response
$R_{\mathrm{sta}}^{\mathrm{na}}$	static response, negative region, after the vehicle leaves the span where the instrument is located
$R_{\mathrm{sta}}^{\mathrm{nb}}$	static response, negative region, before the vehicle reaches the span where the instrument is located
$R_{\mathrm{sta}}^{\mathrm{p}}$	static response, positive region
R_{fil}	filtered static response
$R_{\mathrm{fil}}^{\mathrm{dyn}}$	filtered static response at time of maximum dynamic response
s	separation factor for dynamic loading
S_{p}	speed parameter
T	fundamental period of the bridge
v	vehicle speed
W_i	load for axle number i
w_{t}	width of truck
α	defines the influence zone for maximum static response
α_{L}	live load factor
β	safety index